جامعة الملك عبدالله للعلوم والتقنية

King Abdullah University of Science and Technology

# AMCS 250P

## Applied Linear Algebra and Vector Calculus

## Course description

This course introduces fundamental concepts in Linear Algebra and Vector Calculus, both of which are essential to machine learning and cybersecurity.

The Linear Algebra section introduces matrices and matrix operations, followed by an exploration of fundamental subspaces. It then covers orthogonal projections and their application in least squares approximation for data fitting. The course also delves into key matrix factorizations, including QR decomposition and eigendecomposition, leading to the study of eigenvalues and eigenvectors. This section concludes with matrix diagonalization, a crucial tool in machine learning and encryption algorithms.

The second section offers a practical introduction to Vector Calculus, focusing on an intuitive, application-driven approach to equip students with the necessary tools for an operational understanding of AI models and cybersecurity analytics. It starts exploring vector-valued functions, emphasizing their geometric interpretation and differentiation. The course then introduces partial derivatives as a measure of change and discusses the chain rule. Finally, this section examines the role of the gradient and the Hessian matrix in optimization algorithms. Particular focus is given to gradient descent, a fundamental technique in both machine learning and cybersecurity.

## Logistics

The course is taught over a total of 8 days, in two sessions of four consecutive days. Linear Algebra will be covered in one session, while Vector Calculus will be covered in the other. Course notes, slides and homework will be posted on Blackboard.

## Learning outcomes

By the end of the course, the students will be able to:

- Perform Gaussian elimination to solve systems of linear equations.
- Identify bases for the fundamental subspaces of matrices.
- Use orthogonal projections in least squares approximation problems
- Apply the Gram-Schmidt process to orthogonalize a set of vectors, perform QR decomposition, and use it to solve linear systems and least squares problems.
- Determine the eigenvalues and eigenvectors of a matrix, perform eigendecomposition when applicable, and apply it to Markov matrices.
- Work with vector-valued functions and determine the tangent vector of a space curve at a given point, when it exists.
- Interpret partial derivatives and compute the gradient of multivariable functions.

- Compute rates of change using the chain rule.
- Use the gradient and Hessian matrix in the context of optimization problems.
- Explain and apply the gradient descent algorithm.

## Schedule

- Matrices, matrix operations, and solving a system of linear equations
- The fundamental subspaces of a matrix
- Least squares approximation, QR factorization, and the Gram-Schmidt process.
- Eigendecomposition.
- Vector-valued functions.
- Partial derivatives and the gradient.
- Chain rule.
- Critical points and extrema.
- Gradient-descent algorithm.

## Grading

The course grade will be based on:

- 2 sets of homework (20% each). Keeping up with the homework is the best way to make progress in the course.
- One exam divided into two sections: Linear Algebra and Calculus, each contributing 30% to the total grade.

## Resources

- Course notes and presentation slides will be posted on Blackboard.
- Introduction to Linear Algebra by Glibert Strang, $5^{th}$ edition.
- Computational Science and Engineering by Gilbert Strang.
- Introduction to Linear Algebra by Defranza and Gagrialdi.
- Advanced Engineering Mathematics by Dennis Zill, $7^{th}$ edition (any other edition ok).
- Calculus (Early Transcendentals) by James Stewart, $7^{th}$ edition (any other edition ok).